$L=\frac{CS}{2}$ and $l=\frac{cs}{2}$. $\therefore L-l=\frac{CS-cs}{2}$.

$\frac{S}{s}=\frac{R}{r}=\frac{C}{c}$. $\therefore Sc=Cs$.

$\therefore Sc-Cs=0$. $CS-cs=CS-cs$.

$\therefore CS+Sc-Cs-cs=CS-cs$. $\therefore (C+c)(S-s)=CS-cs$.

$\therefore L-l=\frac{(S-s)(C+c)}{2}$. Q. E. D.

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

Complete the frustum to a full cone, and by cutting along an element spread out the surface into a plane, which can be done with all developable surfaces. The circular arc AB is equal to the circumference of one of the base circles of the frustum $=2\pi R$, and arc $CD=2\pi r$.

$AC=BD=$slant height l of frustum.

From $OA:OC=AB:CD=R:r$, and $OA-OC=l$, we get $OA=\frac{Rl}{R-r}$, $OC=\frac{rl}{R-r}$.

Area of $ABCD=\frac{1}{2}AB.OA-\frac{1}{2}CD.OC=\frac{1}{2}.2\pi R.\frac{Rl}{R-r}-\frac{1}{2}.2\pi r.\frac{rl}{R-r}=\pi l.\frac{R^2-r^2}{R-r}=\pi l(R+r)$.

CALCULUS.

277. **Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.**

Find $\frac{d^2x}{ds^2}$ and $\frac{d^2y}{ds^2}$ for $y=c\sinh\frac{x}{c}$.

Solution by J, W. CLAWSON, Collegeville, Pa.

$y=c\sinh\frac{x}{c}$. $\therefore\frac{dy}{dx}=\cosh\frac{x}{c}$. But $ds^2=dx^2+dy^2$.

$\therefore\left(\frac{ds}{dx}\right)^2=1+\cosh^2\frac{x}{c}$ and $\left(\frac{ds}{dy}\right)^2=1+\frac{1}{\cosh^2(x/c)}$.

$\therefore\frac{dx}{ds}=(1+\cosh^2\frac{x}{c})^{-\frac{1}{2}}$ and $\frac{dy}{ds}=(1+\cosh^2\frac{x}{c})^{-\frac{1}{2}}.\cosh\frac{x}{c}$.

$\therefore\frac{d^2x}{ds^2}=-\frac{1}{2}(1+\cosh^2\frac{x}{c})^{-\frac{3}{2}}.\ 2\cosh\frac{x}{c}\sinh\frac{x}{c}.\frac{1}{c}.(1+\cosh^2\frac{x}{c})^{-\frac{1}{2}}$

$$=-\frac{1}{c}\frac{\sinh(x/c)\ \cosh(x/c)}{[1+\cosh^2(x/c)]^2}.$$

$$\frac{d^2y}{ds^2}=-\tfrac{1}{2}(1+\cosh^2\frac{x}{c})^{-\frac{3}{2}}.2\cosh\frac{x}{c}\sinh\frac{x}{c}.\frac{1}{c}.\ (1+\cosh^2\frac{x}{c})^{-\frac{1}{2}}.\cosh\frac{x}{c}$$

$$+\sinh\frac{x}{c}.\frac{1}{c}(1+\cos^2\frac{x}{c})^{-\frac{1}{2}}.(1+\cosh^2\frac{x}{c})^{-\frac{1}{2}}=\frac{1}{c}\frac{\sinh(x/c)}{[1+\cosh^2(x/c)]^2}.$$

Also solved by H. C. Feemster, G. B. M. Zerr, V. M. Spunar, and J. Scheffer.

278. Proposed by S. A. COREY, Hiteman, Iowa.

If C be Euler's constant, .577,215,664,9... and if B_1, B_2, B_3, etc., be Bernoulli's numbers, $\frac{1}{6}$, $\frac{1}{30}$, $\frac{1}{42}$, etc., prove that

$$C=\tfrac{1}{2}+\frac{B_1}{2}-\frac{B_2}{4}+\frac{B_3}{6}-\frac{B_4}{8}+\ldots-(1)^m\frac{B_m}{2m}+\ldots.$$

I. Solution by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Euler's constant C may be presented under various forms from among which an elementary one may be the following:

$$C=\underset{n\doteq\infty}{\text{Lim.}}[1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\ldots\frac{1}{n}-\log n]=.577,215,664,9\ldots$$

By Taylor's Theorem, we have

$$f(a+h)-f(a)=hf'(a)+\frac{h^2}{2!}f''(a)+\frac{h^3}{3!}f'''(a)+\ldots+\frac{h^n}{n!}f^{(n)}(a)+\ldots$$

Change a successively into $a+h$, $a+2h$, ..., $a+(n-1)h$ and add, then if we put x for $(a+nh)$, we obtain the following result:

$$f(x)-f(a)=h\,\Sigma f'(x)+\frac{h^2}{2!}\Sigma f''(x)+\frac{h^3}{3!}\Sigma f'''(x)+\ldots+\frac{h^3}{n!}\Sigma f^{(n)}(x)+\ldots$$

where $\Sigma f'(x)=f'(x)+f'(a+h)+\ldots$, $\Sigma f''(x)=f''(a)+f''(a+h)+\ldots$, etc.

Let $\phi(x)=f'(x)$. Then

$$\int_a^{a+nh}\phi(x)dx=h\,\Sigma\phi(x)+\frac{h^2}{2!}\Sigma\phi'(x)+\frac{h^3}{3!}\Sigma\phi''(x)+\ldots+\frac{h^n}{n!}\Sigma\phi^{(n-1)}(x)+\ldots.$$